

Exercise 36

- (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.
- (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

Solution

Start by finding the slope of y at $x = 3$. Evaluate the derivative using the quotient rule.

$$\begin{aligned} y' &= \frac{d}{dx} \left(\frac{x}{1 + x^2} \right) \\ &= \frac{\left[\frac{d}{dx}(x) \right] (1 + x^2) - \left[\frac{d}{dx}(1 + x^2) \right] (x)}{(1 + x^2)^2} \\ &= \frac{(1)(1 + x^2) - (2x)(x)}{(1 + x^2)^2} \\ &= \frac{1 - x^2}{(1 + x^2)^2} \end{aligned}$$

Evaluate it at $x = 3$.

$$y'(3) = -\frac{2}{25}$$

Therefore, the equation of the tangent line with slope $-2/25$ that goes through $(3, 0.3)$ is

$$y - 0.3 = -\frac{2}{25}(x - 3).$$

